Modelling of Nonlinear Pulse Propagation in Coupled Microring Resonators

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ABSTRACT
We demonstrate a finite-difference scheme for solution of nonlinear coupled evolution equations that describe propagation of optical pulses under the slowly-varying envelope approximation. The technique is used for modelling of Kerr-nonlinear structures which involve microring resonators and exhibit optical bistability and self-pulsing. The results suggest that the technique may be considered as a useful counterpart of the established methods, such as the transfer matrix method or finite-difference time-domain method.

Keywords: ring resonator, nonlinear optics, bistability, self-pulsing, numerical modelling, integrated optics, coupled mode theory.

1. INTRODUCTION
Computational methods for simulation of nonlinear light propagation are of fundamental importance in the analysis and design of new functional devices. Considering coupled microring structures, theoretical studies of their nonlinear properties are often based on a nonlinear variant of the transfer matrix method (TMM) [1,2,3]. However, the method requires a solution of a nonlinear matrix problem, which may not always quickly converge nor be unique, and provides results in the frequency domain only. The latter can limit the usefulness of the technique. For example, the TMM cannot describe dynamics of optical bistability, a phenomenon, which is expected to play an important role in data processing applications [4]. Coupled resonant structures may also exhibit further interesting effects, such as generation of optical pulses from continuous wave input (self-pulsing) [5], which cannot be simulated in the frequency domain at all. These constraints can be overcome by using the finite-difference time-domain (FD-TD) method. However, for an exact description of resonant structures it is necessary to use very high spatial resolutions, resulting in time-consuming calculation, and/or apply advanced algorithms with correction of the phase velocity error [6,5].

Recently, we presented a simple numerical technique that avoids some of the mentioned problems [7]. Under the slowly-varying envelope approximation, propagation of optical pulses in coupled microring systems is described by a system of coupled partial differential equations. These equations are solved by an explicit finite-difference scheme based on upwind differencing. We will denote the technique as CE (coupled equations). We have presented stability criterions and comparison with the TMM. To this aim the CE technique has been applied to Kerr-nonlinear structure consisting of one ring coupled to a waveguide.

Here, we present simulation of more complex structures, nonlinear optical channel dropping filters, which include 1 and 3 microrings. The structures exhibit optical bistability and self-pulsing and were chosen with the aim to demonstrate typical circumstances in which the CE method may be useful.

2. FORMULATION
We suppose that the whole structure, such as channel dropping filter shown in Fig. 1, is composed of two kinds of elements: single waveguides and waveguide couplers. The CE method simulates propagation of waves in each element and simultaneously uses boundary condition between adjacent elements. Consider for example coupler in dashed box in Fig. 1. $f^{(i)}(Z,T)$ and $f^{(i+1)}(Z,T)$ represent the dimensionless mode field envelopes in the waveguides, $Z = z\frac{2\pi}{\lambda}$ and $T = t\frac{2\pi c}{\lambda}$ are the normalized spatial and time variables, $L^{(i)}_c$ is the normalized coupling length and $\kappa^{(i)}(Z)$ is the normalized coupling coefficient. We assume that the waveguides are single mode, the effective mode indices are supposed to be equal to the mode group indices $n^{(i)}_g$ and $n^{(i+1)}_g$.

One of main advantages of the CE scheme is that it can deal with various nonlinear effects. Here, we suppose instantaneous Kerr-nonlinearity. The (dimensionless) nonlinear Kerr indices in the waveguides are denoted by $n^{(i)}_t$ and $n^{(i+1)}_t$.

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Propagation of optical pulses under the slowly-varying envelope approximation is given by

\[
\frac{\partial f^{(i)}}{\partial z} + n_g^{(i)} \frac{\partial f^{(i)}}{\partial T} = i\kappa^{(i)}(Z)f^{(i+1)} + \left| f^{(i)} \right|^2 f^{(i)},
\]

(1)

\[
\frac{\partial f^{(i+1)}}{\partial z} + n_g^{(i+1)} \frac{\partial f^{(i+1)}}{\partial T} = i\kappa^{(i)}(Z)f^{(i)} + \left| f^{(i+1)} \right|^2 f^{(i+1)},
\]

(2)

For simplicity, value of \( \kappa^{(i)}(Z) \) is supposed to be constant and given by the relation

\[
s^{(i)} = \sin \left( \kappa^{(i)} L \right),
\]

(3)

where \( s^{(i)} \) is imaginary part of the net coupling coefficient (i.e. \( \kappa^{(i)} \) is relative field amplitude coupled across the coupler in the steady-state). Equation (3) follows from steady-state solution of (1) and (2) in the linear case.

The coupled system (1) and (2) is discretized and solved by using the explicit finite-difference scheme described in [7]. Similar approach is taken for all elements in the structure. Boundary conditions, which are used during the calculation, are given by the continuity of the field and also by field envelope at the input port (labeled as In in Fig. 1).

3. NUMERICAL RESULTS

3.1 Bistability

The first example is channel dropping filter consisting of single ring with circumference \( L = 31\pi \mu m \). The structure includes two identical couplers described by parameters: \( s^{(0)} = s^{(1)} = 0.1 \) and \( \kappa_0^{(0)} = \kappa_0^{(1)} = 4 \mu m \). Mode effective indices, \( n_g^{(0)} = 1.6 \), and Kerr-indices are the same in all waveguides.

![Figure 2](image-url). Dependence of normalized intensity at through port (a) and drop port (b) on normalized input intensity. Solid lines represent stable solutions obtained by CE, dotted lines represent unstable solutions calculated by TMM. The calculation parameters are described in the text.
Figure 3. Time dependence of normalized intensity at input port, through port and drop port.

The structure parameters and wavelength are as in Fig. 2.

Figure 2 presents steady-state solutions obtained by the CE method and a comparison with the TMM. The nonlinear response was calculated for wavelength \( \lambda = 1.54287 \) µm (above the resonance \( \lambda_r = 1.5428 \) µm). \( n_z I_{in} \) is the normalized input intensity (i.e. the nonlinear change of the effective index at the input port), \( n_z I_{through} \) and \( n_z I_{drop} \) have similar meaning. In the bistable region, the TMM provides 3 solutions (stable solutions are identical with CE results) while the CE technique naturally converges to the stable solutions only.

Figure 3 shows all-optical switching functionality. The input intensity \( n_z I_{in} = 9 \times 10^{-5} \) was modulated with positive and negative pulses with duration about 400 ps.

3.2 Self-pulsing

The second example is the device with 3 identical rings. Ring circumferences are \( L = (7.0 + 13 \pi) \) µm. The coupling coefficients, which are optimized for flat response, \( s^{(0)} = s^{(3)} = 0.4668 \) and \( s^{(1)} = s^{(2)} = 0.099 \), are taken from [1]. All coupling lengths are \( L^{(i)} = 3.5 \) µm. Mode effective indices, \( n_{e}^{(0)} = 4.25 \), and Kerr-indices are the same in all waveguides.

Figure 4 shows spectral dependencies of drop port transmission and group delay near resonance wavelength \( \lambda_r = 1.5640 \) µm for various levels of nonlinearity. As expected, nonlinearity shifts spectra towards longer wavelengths and group delay is increased near band edge for \( \lambda > \lambda_r \). Moreover, TMM calculation [dashed lines in Fig. 4 (a); these solutions fulfill stability condition \( dI_{drop} / dI_{in} > 0 \)] suggests that bistability may occur for \( \lambda > \lambda_r \).

This conclusion, however, is not confirmed by CE method, which reveals unstable solutions that exhibit self-pulsing behaviour. These solutions are shown in Fig. 5. Note, that shape of the pulses, their amplitude and period depend only on the normalized input intensity in the steady state and detuning from resonance wavelength. For example, as illustrated in Fig. 5, period of the pulses decreases with increasing nonlinearity. The phenomenon will be subject of further investigations.
4. CONCLUSIONS

In summary we have presented a simple finite-difference scheme for solution of nonlinear coupled evolution equations that describe propagation of optical pulses in Kerr-nonlinear structures. The technique has been used for simulation of nonlinear optical channel dropping filters with 1 and 3 coupled microrings.

Generalization of the technique to the more complex systems is straightforward. One of main advantages of the scheme is that it enables easy inclusion of various nonlinear effects. The presented examples suggest that the technique may be considered as a useful counterpart of the established methods, such as the TMM or FD-TD.

Figure 5. Time dependence of normalized intensity at input port, through port and drop port for two levels of normalized input intensity in steady state \( \eta_2 I_n = 5 \times 10^5 \) (a) and \( \eta_2 I_n = 10^4 \) (b). The structure parameters are as in Fig. 4, detuning from resonance wavelength is 0.5 nm.

REFERENCES