Simple numerical scheme for modelling of nonlinear pulse propagation in coupled microring resonators

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ABSTRACT
Nonlinear coupled equations that describe propagation of optical pulses in coupled microring resonators under the slowly varying envelope approximation can be solved by a simple explicit finite-difference scheme. The technique, which is based on upwind differencing, is demonstrated by using an example of Kerr-nonlinear structure consisting of one microring resonator coupled to a waveguide. This enables us to study stability of the scheme and its typical features.

Keywords: ring resonator, nonlinear optics, bistability, numerical modelling, integrated optics, coupled mode theory

1. INTRODUCTION
One of the most attractive features of coupled resonant structures is that they enhance nonlinear processes [1,2] without affecting bandwidth [3]. In case of coupled microrings, theoretical studies of their nonlinear properties are often based on a nonlinear variant of the transfer matrix method (TMM) [4,5,6]. However, the method requires a solution of a nonlinear matrix problem, which may not always quickly converge nor be unique, and provides results in the frequency domain only. The latter can limit the usefulness of the technique. For example, the TMM cannot describe dynamics of optical bistability, a phenomenon, which is expected to play an important role in data processing applications [7]. Coupled resonant structures may also exhibit further interesting effects, such as generation of optical pulses from continuous wave input [8], which cannot be simulated in the frequency domain at all. These constraints can be overcome by using the finite-difference time-domain (FD-TD) method. However, for an exact description of resonant structures it is necessary to use very high spatial resolutions, resulting in time-consuming calculation, and/or apply advanced algorithms with correction of the phase velocity error [9,8].

Here, we present a simple numerical technique that avoids the mentioned problems. Under the slowly varying envelope approximation, propagation of optical pulses in coupled microring systems is described by a system of coupled partial differential equations. These equations can be solved by a simple explicit finite-difference scheme based on upwind differencing. In principle, the scheme can be used for various nonlinear phenomena and systems with more coupled rings. The aim of this paper is to present the technique using a simple example of Kerr-nonlinear structure consisting of one microring side-coupled to a waveguide. This enables us to study stability of the scheme and compare its performance with the TMM.

2. FORMULATION
Consider a structure as shown in Fig. 1. We assume that the ring and the waveguide are single mode with the same mode effective index which is equal to the mode group index $n_g$. Let $f(Z,T)$ and $g(Z,T)$ represent the dimensionless mode field envelope in the waveguide and the ring respectively, $Z = z \frac{2\pi}{\lambda}$ and $T = t \frac{2\pi}{\lambda}$ are the normalized spatial and time variables, $L$ is the normalized coupling length, and $L_R$ is the normalized ring circumference.

Figure 1. A racetrack microring resonator side-coupled to a waveguide. $Z$ denotes the normalized coordinate along the ring and the waveguide. Both coordinates are identical in the coupling region.

Propagation of optical pulses under the slowly varying envelope approximation is given by
In these equations $\kappa(Z)$ is the normalized coupling coefficient ($\kappa(Z) = 0$ outside of the coupling region), and $n_z$ is the (dimensionless) nonlinear Kerr index. For simplicity, value of $\kappa(Z)$ inside of the coupling region is supposed to be constant and given by the relation

$$s = \sin(kL)$$

where $s$ is imaginary part of the net coupling coefficient (i.e. $is$ is relative field amplitude coupled across the coupler in the steady-state). Equation (3) follows from steady-state solution of (1) and (2) in the linear case.

In order to solve equations (1) and (2), it is necessary to know boundary conditions. For the structure in Fig. 1, they are given by function $f(Z = 0, T)$, which describes the field envelope at the waveguide input, and by condition $g(Z = 0, T) = g(Z = L_n, T) \exp(i n_x L_n)$, which follows from the continuity of the field in the ring.

The coupled system (1) and (2) is discretized on uniform grid with spacing $\Delta Z$ and $\Delta T$ and solved by using the explicit finite-difference scheme

$$f_j^{n+1} - f_j^n = -\alpha(f_j^n - f_{j+1}^n) + i \gamma(g_j^n + g_{j+1}^n) + i \beta\left(f_j^n + f_{j+1}^n\right)^2 \left(f_j^n + f_{j+1}^n\right)$$

$$g_j^{n+1} - g_j^n = -\alpha(g_j^n - g_{j+1}^n) + i \gamma(f_j^n + f_{j+1}^n) + i \beta\left(g_j^n + g_{j+1}^n\right)^2 \left(g_j^n + g_{j+1}^n\right)$$

where $f_j^n = f(j \Delta Z, n \Delta T)$, $g_j^n = g(j \Delta Z, n \Delta T)$, $j = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$, $\alpha = \frac{\Delta T}{n_x \Delta Z}$, $\gamma = \frac{\kappa \Delta T}{2 n_x}$, and $\beta = \frac{n_z \Delta T}{8 n_x}$.

The first term on the RHS of equation (4) or (5) stems from upwind differencing of the derivatives on the LHS of (3) or (4) [10]. It is well known that the upwind scheme (i.e. here for $\gamma = \beta = 0$) is numerically stable if the Courant condition

$$\alpha \leq 1$$

is satisfied.

The form of the discrete representation of the RHS of (1) or (2) follows from our numerical experiments with various alternatives. The presented formulation, which use average values [see the second- and third term on the RHS of equations (4) or (5)], provided the most accurate and stable results.

In order to check the numerical stability of the proposed scheme, we linearized equations (4) and (5) and applied von Neumann stability analysis [10]. It results in the Courant condition (6) again accompanied by an additional criterion

$$\gamma + C \leq \sqrt{\frac{1 - \alpha}{2}}$$

where $C = \frac{n_z \Delta T}{2 n_x} I_{\text{max}}$ and $I_{\text{max}}$ is maximum of values $\left|f_j^n\right|^2$ and $\left|g_j^n\right|^2$. The last condition, which is illustrated in Fig. 2(a), can be rewritten into the form

$$\left[\arcsin(s) + n_z L_{\text{max}}\right] \Delta Z \leq \sqrt{\frac{2(1 - \alpha)}{\alpha}}$$

Note, that $n_z I_{\text{max}}$ is maximum of nonlinear change of effective index over the structure. In typical calculation, $n_z I_{\text{max}} < 10^{-3}$ (the upper limit for most transparent materials is of the order of $10^{-2}$ [1]), $\alpha = 0.9$, $\Delta Z < L$, and $\Delta Z < 2\pi$. Thus, for small $s$ and/or for long $L$ inequality (8) is well satisfied. The opposite case (large $s$ and short $L$) needs more attention; $n_z I_{\text{max}}$ can be neglected and the condition (8) takes the form

$$\frac{L}{\Delta Z} \geq \arcsin(s) \frac{\alpha}{\sqrt{2(1 - \alpha)}}$$

which is illustrated in Fig. 2(b).
3. NUMERICAL EXAMPLES

To demonstrate the developed technique we simulated the structure in Fig. 1. The first example presents steady-state solutions of equations (1) and (2) and a comparison with the TMM. The device is fully transmissive, so that the ring resonator simply causes an additional phase shift of the output field. Fig. 3(a) shows the shift near the resonance $\lambda = 1.5 \, \mu m$, $n_2 I_{in}$ is the normalized input intensity (i.e. the nonlinear change of the effective index at the waveguide input). As expected, nonlinearity enhances the phase shift and bistability may occur for $\lambda > \lambda$.

Both numerical techniques yield the same results. Different features can be seen in Fig. 3(b). TMM calculation starts with the normalized intensity in ring $n_2 I_{out}$ (vertical axis) and returns the phase and the normalized intensity at the input and output. Solution for given input intensity must be obtained numerically. In the bistable region this approach provides 3 solutions [only stable solutions are shown in Fig. 3(a)]. The presented technique naturally converges to the stable solution only. The result depends on the input intensity in the steady-state and, in the case of bistability, on the history of the input signal as well.

The second example (see Fig. 4) demonstrates the capability of the presented technique to simulate propagation of optical pulses. The input pulse is centred at the wavelength $\lambda = 1.5002 \, \mu m$ (above the resonance). Temporal delay of the output pulse in the linear case (5.2 ps) is the same as group delay calculated by the TMM. With increasing level of nonlinearity the resonator is tuned into resonance and the delay approaches 20 ps, which is the value of linear group delay calculated by the TMM for the resonance wavelength. Further increasing of nonlinearity causes severe distortion of the pulse accompanied by a gradual reduction of the delay. Finally, the pulse propagates without any ring-induced delay (not shown in the Fig. 4).
Figure 4. Intensity profiles of the input and the output pulse for various levels of nonlinearity. Structure parameters are as in Fig. 3.

4. CONCLUSIONS

In summary we have presented a simple finite-difference scheme for solution of nonlinear coupled evolution equations that describe propagation of optical pulses under the slowly-varying envelope approximation. The technique, which is based on upwind differencing, has been applied to Kerr-nonlinear structure consisting of one ring coupled to a waveguide. We have presented stability criterions and comparison with the TMM. Generalization of the technique to the systems with more coupled rings and/or waveguides is straightforward. One of main advantages of the scheme is that it enables easy inclusion of various nonlinear effects. The presented examples suggest that the technique may be considered as a useful counterpart of the established methods, such as the TMM or FD-TD, namely when studying dynamics of nonlinear propagation in the resonant structures.

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